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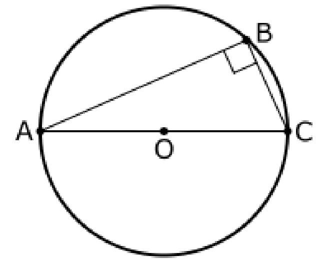
MTH 4040: Coordinating Seminar

Proofs of Thales' Theorem in GeoGebra
High School Level Geometry



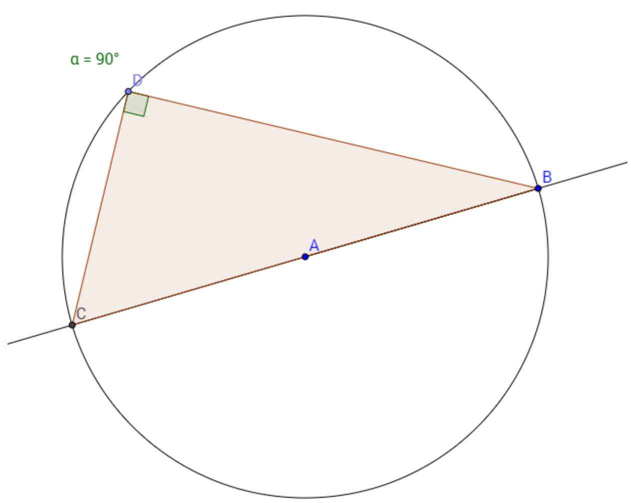
Thales' Theorem:
If *A*, *B* and *C* are points on a circle where the line *AC* is a diameter of the circle, then the angle $\angle ABC$ is a right angle.

Geometric Proof:

1) Go to the online version of GeoGebra at "http://web.geogebra.org/app/#".
2) Select Geometry mode.
3) Construct segment *AB*.
4) Construct a circle with center at point *A* and going through point *B*.
5) Construct a line going through points *A* and *B*.
6) Construct point *C* at the intersection of the line and the circle, forming the diameter of the circle.
7) Construct point *D* on the circle.
8) Create $\triangle BCD$ with the polygon tool .
9) Create an interior angle of triangle *BCD* at point *D*.
10) Drag point D along the circle. What do you notice?

How can we come up with a geometric proof for this theorem?

Hint: Construct the radius *AD* as a segment .



Trigonometric Proof:

1) Go to the online version of GeoGebra at "http://web.geogebra.org/app/#".
2) Select Algebra mode.
3) Input "O=(0,0)" to create point *O*
4) Input "A=(-1,0)" to create point *A*
5) Input "C=(1,0)" to create point *C*
6) Construct a circle with center at point *O* and going through points *A* and *C*.
7) Input "B=(cosθ, sinθ)" to create point *B* and a slider for θ
8) Create △*ABC* with the polygon tool .

We have trigonometrically constructed the model for Thales' Theorem.

Now, to prove this theorem we will show that △*ABC* forms right angles by proving that lines *AB* and *BC* are perpendicular. We can show this by finding the product of both lines. If the product is -1, then the lines are perpendicular.

9) Let's define M_1 as the slope of line *AB*.

$$M_1 = \frac{Y_B - Y_A}{X_B - X_A} = \frac{\sin\Theta - 0}{\cos\Theta + 1} = \frac{\sin\Theta}{\cos\Theta + 1}$$

Input "M_1=(sin(Θ))/(cos(Θ)+1)," so that GeoGebra will define M_1 for every value of Θ.

10) Let's define M_2 as the slope of line *BC*.

$$M_2 = \frac{Y_B - Y_C}{X_B - X_C} = \frac{\sin\Theta - 0}{\cos\Theta - 1} = \frac{\sin\Theta}{\cos\Theta - 1}$$

Input "M_2=(sin(Θ))/(cos(Θ)-1)," so that GeoGebra will define M_2 for every value of Θ.

11) Let's define *P* as the product of the slopes of lines *AB* and *BC*.
Input "P=(M_1)(M_2)," so that GeoGebra will define *P* for every value of M_1 and M_2.

Move the slider for Θ from left to right. What do you notice about the value of *P*?

For every value of Θ, the value of *P* is always -1. This means that for every position of point *B* in the circle, the angle formed at point *B* is always a 90° angle.

This is because:

$$M_1 \cdot M_2 = \frac{\sin\Theta}{\cos\Theta + 1} \cdot \frac{\sin\Theta}{\cos\Theta - 1} = \frac{\sin^2\Theta}{\cos^2 - 1} = \frac{\sin^2\Theta}{-\sin^2\Theta} = -1$$

Therefore, *P* is always -1 for every value of Θ and ∠*ABC* is always a right angle.